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School Textbooks as a Medium for the Intellectual Development of Children During the Mathematics Teaching Process

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Summary: One way of developing students' intelligence is through the design of new school textbooks and educational material of a type that meets the requirements of a psycho-didactic approach. As part of the "Mathematics, Psychology, Intelligence" education project (MPI), mathematics textbooks and educational material (study books, practical work, workbooks for independent study, computer software) for middle school pupils (Years 5 to 9) were developed within the framework of an "enrichment" teaching model. The basic purpose of this model is the intellectual development of students through their mathematical education using specially constructed educational texts. The specific nature of these educational texts lies in the fact that, at the same time as conveying structures of formal mathematical knowledge, they also (1) support development of the basic components of students' mental experience (including cognitive, conceptual, metacognitive and intentional experience) and (2) create conditions for students to employ their own individual cognitive styles.

Key words: Mathematics education, psycho-didactics, students' intellectual development, educational texts.

Introduction

As a result of contemporary social challenges, people's intellectual abilities are beginning to be considered a key factor in the progressive development of society. Therefore, the task of shaping the intellectual resources of the rising generation within a comprehensive school education framework relates to a number of national priorities. A high level of intellectual resources in school graduates lays the foundation for willingness to engage in innovative activity in subsequent stages of their education and professional careers.

Tackling the issue of schoolchildren's intellectual development means taking account of two basic conditions: we need first to review criteria used to evaluate the effectiveness of the educational process, and second to develop new teaching technologies. In other words, developing

willingness to engage in innovative activity presupposes the introduction of innovative forms and methods of teaching.

Criteria to assess the effectiveness of the teaching process are, in our view, the following *intellectual qualities* of the schoolchild. We consider these to be preconditions for willingness to engage in innovative activity (CICSU):

C – Competence. Intellectual competence means those particular features of knowledge organisation which ensure the ability to take effective decisions in specific spheres of activity, including diversity, structuredness, flexibility, efficiency, trans-situationality, categorical character, unity of declarative and procedural knowledge, understanding of one's own knowledge.
I – Initiative. Intellectual initiative is the desire to discover new information, to advance ideas and to master different spheres of activity independently and of one's own accord.
C – Creativity. Intellectual creativity is the process of creating what is subjectively or objectively new, based on the ability to generate original ideas, to use non-standard methods and show tolerance towards unusual and "impossible" situations.
S – Self-regulation. Intellectual self-regulation is the ability to control voluntarily one's own intellectual activity and purposively construct a process of self-teaching.
U – Uniqueness of mind-set. Uniqueness of mind-set is the individually distinct mode of cognitive relation to immediate outside occurrences, including the expression of individual cognitive styles and the extent to which individual cognitive preferences are formed.

It is possible to shape these intellectual qualities in an individual within the framework of innovative teaching technologies developed on the basis of the *psycho-didactic approach*.

The psycho-didactic approach as an alternative to the subject-centred approach in school education

In the traditional system of teaching, constructed on the basis of the *subject-centred* approach, criteria for judging the effectiveness of students' education are mainly associated with the level of knowledge awareness, abilities and skills obtained from each school subject. Within innovative teaching technologies developed on the basis of the *psycho-didactic* approach, the emphasis fundamentally shifts: the criteria for evaluating the effectiveness of the educational process become those changes in the student's intellectual and personal realm which characterise his or her development as a productive, self-sufficient and active individual.

Psycho-didactics is the area of pedagogy in which content, forms and methods of teaching are designed based on the integration of psychological, didactic, methodological and thematic (corresponding to different school subjects) knowledge, with priority given to using patterns of psychic personality development as a basis for organising the teaching process and general educational environment (Davydov 1966; Panov 2007; Gelfman and Kholodnaya 2006; Gelfman *et al*. 1997; Kidron *et al*. 2010; Brousseau 1997; Malara and Navarra 2003; and others).

The result of psycho-didactic work is a qualitatively new pedagogical product combining psychological, didactic, methodological and thematic knowledge, in the form of a new type of school environment, innovative educational technologies, a developmental method of education, a new generation of school textbooks and so on. At the base of the psycho-didactic approach in its direct sense lies pedagogical engineering; that is, the process of designing, constructing and exploiting

pedagogical products, inherently oriented towards developing the mental resources of each schoolchild. The basic purpose of psycho-didactics is to create the conditions for students' psychological growth on the basis of improving teaching effectiveness in a given school subject.

There are various routes to implementing the psycho-didactic approach in school education: using "didactic situations" which build up students' knowledge during the learning process, including the use of metaphor and emotional context (Broussau 1997); an orientation towards comprehension of the learning material and formation of ideas by selecting mathematical tasks and hypotheses on the basis of their influence on the process of education ("Hypothetical Learning Trajectory" – HLT) (Simon 1995; Simon and Tzur 2004); the development of basic cognitive operations, such as identification, combination and construction, as the basis of conceptual teaching (the RBC model) supported by the personal experience of the student (Hershkowitz, Schwarz and Dreyfus 2001; Bikner-Ahsbahs 2004); the development of students' creative thought skills (Burke and Williams 2008) and others. In our view, the key direction in psycho-didactics that guarantees an innovational teaching regime is the *psychologically grounded design of the content of school education*.

Accordingly, the question of the requirements for school textbooks is particularly relevant. Within the traditional subject-centred approach the purpose of the mathematics textbook is reduced to a strict, sequential account of established mathematical knowledge adapted to the age of the students. The form these traditional textbooks take is of a reference book and a workbook of mathematical problems. The implication is that the teaching of mathematics should be carried out by means of mathematics itself, since mathematical knowledge in and of itself possesses an essential developmental effect.

From the point of view of the psycho-didactic approach, the new generation school textbook is a *polyfunctional psychological system*, realising the goals of an array of new functions (Gelfman and Kholodnaya 2006).

1 The informational function (informing students about the various areas of academic knowledge, taking into account: the accessibility of textbook information; a specific form of structuring information, in the sense of the relationship between reference, explanatory and problem-based texts; an orientation towards comprehension of educational material; a balance of extending and condensing in the text; a unity of declarative and procedural knowledge).
2 The directive function (the availability of instructive information and organisation of "enriching" repetition, where past material is repeated at the same time as assimilating new; the creation of conditions for students' investigative activity; their stimulation towards independent work; inclusion of means for routine diagnostics).
3 The developmental function (the creation of conditions for concept formation; the development of general intellectual skills, including the ability to reason, justify, prove, criticise, take rational decisions and so on; the building up of a reflexive position in the schoolchild; motivation towards educational activity; the development of students' creative facilities).
4 The communicative function (problematisation of an educational text, including using various different kinds of question; its interactive character; the expressive style of its presentation).
5 The educational function (the presence of knowledge relating to a cultural worldview, of methodological and historical-scientific knowledge; an initiation through the text of an evaluating attitude in the students towards educational material; personal relevance of an educational text, taking into account the student's personal experience; an orientation towards the formation of his or her personal qualities).
6 The function of differentiation and individualisation of learning (taking into account through the educational text the students' individual tempos of learning, their individual cognitive styles, their different inclinations and levels of interest in the subject and their opportunities to choose a regime of educational activity).

The question naturally arises: can a textbook fulfil of all these functions? Using a traditional textbook it undoubtedly cannot be done. New generation textbooks are necessary, constructed on a fundamentally different basis. First of all, the development of the textbook's content, structure and form should take into account the *requirements of the psycho-didactic approach*; that is, each element of the textbook (ways of presenting educational information, the sequence and arrangement of the material, the style of presentation and so on) should have a specific psychological target and ensure a definite developmental psychological effect. Second, the textbook should be part of a system of supplementary educational material (study books for pupils, practical work on various types of task, workbooks, computer software and so on) forming an educational and methodical set (EMS), providing a varied and enriched educational space for each school subject.

The next question for discussion then, is how, within the framework of the psycho-didactic approach, is the development of students' intelligence effected?

Enrichment of mental (intellectual) experience as the psychological basis for developing students' intelligence

In our view, the psychological basis of intellectual development is the process of enriching students' mental (intellectual) experience during the education process. Figure 22.1 shows a structural model of intelligence, illustrating its particular organisational features from the point of view of the composition and construction of mental experience (Kholodnaya 2002).

In accordance with the structural model of intelligence put forward, three levels (or layers) of mental experience can be identified, each of which has its own function:

1 *Cognitive experience*. The mental structures allowing storage, regulation and transformation of available and incoming information, facilitating its reproduction in the psyche of the cognisant individual as a stable, ordered aspect of his or her surroundings. Their basic purpose is operative processing of a flow of information about immediate impressions at various levels of cognitive activity.
2 *Metacognitive experience*. The mental structures allowing accomplishment of involuntary and voluntary regulation of information processing, and also conscious management of the work of one's own intelligence. Their basic purpose is to control the state of individual intellectual resources and the course of individual intellectual activity.
3 *Intentional (emotional-evaluative) experience*. The mental structures which lie at the base of individual intellectual inclinations. Their basic role consists in the fact that they predetermine subjective criteria of choice for specific knowledge domains, the direction of decision making, choice of sources of information, means for its representation and so on.

In turn, the particular organisational features of cognitive, conceptual, metacognitive and intentional experience determine the characteristics of the individual's intelligence at the level of productivity of intellectual activity (personal intellectual abilities – convergent and divergent abilities, educability, and stylistic characteristics of intelligence, and also integral intellectual abilities – competence, talent and wisdom as phenomena of intellectual giftedness), and on the level of individual distinctiveness of mind-set (personal cognitive style in the form of a combination of preferences for modes of intellectual activity) (Kholodnaya 2002, 2004).

In identifying intelligence with the particular organisation of individual mental experience, we can say that any pupil is "full" of his or her own mental experience, which will predefine the character of his or her intellectual activity in a given situation. The content and structure of this experience is different in each pupil, therefore children certainly differ in their intellectual resources.

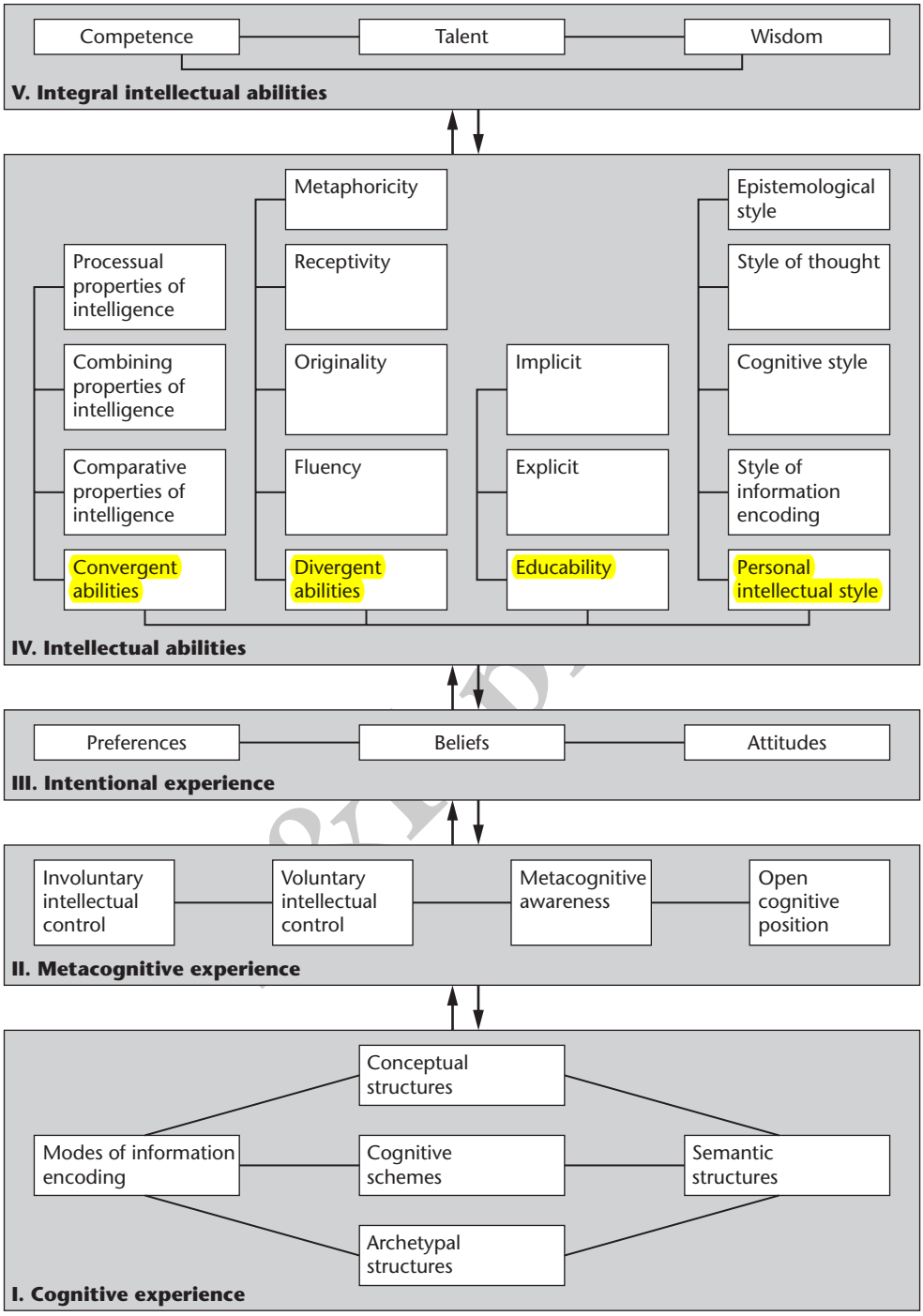

Figure 22.1 A structural model of intelligence illustrating its particular organisational features from the point of view of the composition and construction of mental experience (Kholodnaya 2002).

However, they all need suitable conditions to be created for their intellectual development through the maximum possible enrichment of their individual mental experiences.

"Enrichment" means, first, that the basic components of every student's mental experience are developed (including cognitive, conceptual, metacognitive and intentional experience) as a basis for developing his or her intellectual abilities, including his or her integral intellectual abilities; and, second, that conditions are created to allow students to display their individual cognitive styles and individual cognitive inclinations.

In short, therefore, the essence of the problem of the student's intellectual development can be represented by the following set of propositions:

- Each student is the bearer of mental experience, and consequently possesses certain initial intellectual resources; by virtue of this individual distinctiveness in content and structure of mental experience, each student is "clever in his or her own way".
- The target of pedagogical influence in school education is a distinct set of components of individual mental experience (including its cognitive, conceptual, metacognitive and intentional components).
- The mechanisms for an individual's intellectual growth are connected to processes occurring in the area of individual mental experience and the enrichment and deepening in complexity of all of its components.
- Each student has his or her own range of potential capacity for intellectual strengths, and the task of the teacher lies in providing him or her with the necessary aid by means of individualisation of school and extra-curricular activity.
- Criteria for evaluating the effectiveness of the educational process, alongside knowledge, ability and skill (KAS), are connected with the extent to which the individual's intellectual qualities, such as competence, initiative, creativity, self-regulation and uniqueness of mind-set (CICSU) are formed.

Subsequently, solving the problem of developing students' individual intellectual resources presupposes:

1 Creating conditions for the actualisation of each student's available mental experience (taking into account preferred modes of information encoding, available cognitive schemes, the particularities of the bases of present knowledge, the level of common sense and formed academic understanding, the particularities of intellectual self-regulation, individual cognitive preferences, the individual tempo of learning and so on).
2 The creation of conditions for the increase, enrichment and deepening in complexity of that student's individual mental experience, within its maximum possible limits (conditioning the mind to work with various different modes of information encoding, widening the repertoire of cognitive schemes, differentiation and integration of verbal and non-verbal semantic structures, enrichment of conceptual frameworks, developing the ability to exercise involuntary and voluntary control of one's own intellectual activity, forming open cognitive positions and a high level of metacognitive awareness, creating conditions for mastery of a broad range of different intellectual styles).

Educational texts as a means for students' intellectual development

The key factor influencing the development of students' intelligence is the *content of school education*. In turn, the unit of subject content in schools is the *educational text*.

The text is the most valuable element of culture and the most important constituent of the educational process. Many experts have discussed the special role of the text in the intellectual development of the individual, examining the text as a "thinking structure" (Vyach. vs Ivanov), a "model for thought adventures" (L. E. Gendenshtein) and a "conversation-partner" (M. M. Bakhtin).

Texts (academic, historical-cultural, fictional, educational) are not linear. An educational text should be built as a multidimensional semantic space, within which the pupil-reader can imaginatively roam in different directions. The structure of the educational text, in addition to its "nucleus" (the specific subject information) includes:

- *Context*. Students should have the opportunity to move "horizontally" across the text, expanding their acquaintance with the study material through the use of various forms of presentation of material (verbal, visual, practical, emotional-metaphorical), texts which are heterogeneous in content (stating, explanatory, problem-discursive, multiple-choice), inclusion of off-subject texts (in the form of narrative elements, elements of game situations, applied physics, ecology and psychology material).
- *Subtext*. Students should have the opportunity to move "vertically" up and down the text, which presupposes the segmentation of the text by different stages of complexity, both in content and in the type of activity (the use of texts and tasks of varying levels of difficulty; the inclusion of normative texts with demonstration models of activity and plain texts; education through performance, research, project work or creative activity and so on).
- *Implied text*. Students should have the opportunity to move down "deep" into the text, that is, to draw out the deeper sense of the text: that which is not expressed verbally, through the connotative meaning of words; the content of their own personal experience; associative links and their own imaginations.

In the area of school education interest in texts is connected to an understanding of their role in effective teaching, particularly in the context of *reader-oriented theory*, in which the reader actively constructs meaning (concepts) in the process of reading a text. This includes where a student works with mathematical textbooks (Weinberg and Wiesner 2011).

Within the "Mathematics, Psychology, Intelligence" educational project (MPI), mathematics textbooks and educational material were developed for students at middle school (Years 5–9) on the basis of an "enrichment" model of education. The model's basic purpose is students' intellectual development through the medium of mathematical education by specially constructed educational texts (Gelfman *et al.* 2002; Gelfman and Kholodnaya 2006). Mathematics teaching is carried out using the educational and methodical set (EMS), in keeping with modern pedagogical ideas about the organisation of educational spaces of learning activity.

Included in the MPI project's educational and methodical set are: for Years 5 and 6, textbooks, study books including narrative texts and practical study workbooks, activity books for independent work, electronic educational resources (practice exercises, tests, a library of cartoons, a collection of mathematical games and an electronic reference guide), teachers' books; for Years 7–9, textbooks, books of mathematical problems, teachers' books.

Educational texts for all the elements of the EMS have been developed using the "enrichment" model of mathematics teaching, taking into account the basic positions of activity-based, personality-oriented and competency-based approaches to organising content for modern mathematical education in schools, which represent a concretisation of the psycho-didactic approach.

The *activity-based approach* is implemented in the "enrichment" model whereby the students take an active role in the process of their education in mathematical knowledge and methods for solving mathematical problems:

- The textbook is supplied with navigational tools (a navigation bar with customised icons), which allow the learner to use all of the individual components of the EMS in accordance with his/her educational needs; the navigational icons manage the student's interaction with textbook content and links to other study material (study books, workbooks and electronic resources), involving him or her in various types of educational activity (performance, research, project work, creative work).
- When working with the textbook and educational materials the student is offered the opportunity to exercise independence at various stages of the study of mathematics (for example, in the textbooks work with texts is organised in ways such as an "appeal to the reader", aimed at the initiation of independent activity in the student).
- Conditions are created for the student to develop effective methods of educational-cognitive activity, namely: mastering algorithms, forming an ability to solve text-based problems, developing willingness to choose rational methods of problem-solving and using various types of analysis for the same educational problem and so on.
- In textbooks and study books, alongside statements of "ready-made" mathematical knowledge, the process by which it was derived is shown (new knowledge is introduced gradually, including the stage of motivation to learn a new mathematical concept, the discussion stage, the generalisation stage and conclusions).
- The textbooks and educational material contain applied material, developing an interest in the practical applications of mathematics and demonstrating the role of mathematical knowledge in real situations.

The *personality-oriented approach* to the organisation of educational material is implemented in the "enrichment" model in the following way:

- Educational information is presented in various forms (verbal, visual, practical, emotional-metaphorical), allowing students with varying cognitive styles to grasp the material successfully.
- The active use of students' personal (including day-to-day) experience (both in the stage of mastering the theoretical sections of the textbook and when developing problem solving skills).
- The textbooks and educational materials have a dialogic character that cultivates in the student a readiness to express his or her opinion and to justify and defend his or her point of view.
- Individualised teaching is achieved by means of the textbook and educational material (components of the EMS allow consideration to be given to students' individual cognitive needs and inclinations and allow the selection of an individual trajectory of self-education).

The *competence-based approach* in the "enrichment" model is covered by the following aspects:

- It uses a thematic principle to organise textbooks and educational materials, which simultaneously allows both deepening and broadening of the students' knowledge, and also builds content appropriate to the topic using various types of systematisation of information.
- Simultaneous formation of both declarative knowledge (knowing *that*), and also procedural knowledge (knowing *how*) is planned for.
- The content of textbooks and educational materials (the sequence of study for each topic, selection of questions and educational tasks) is constructed in such a way as to help form a reflexive position (a conscious, volitional attitude of the student towards the process of education).
- The educational material trains schoolchildren in the right way to respond to contradictions.

- Skills of planning, goal-setting, self-monitoring, prediction, evaluation, substantiating and generalisation are developed through the text as the basis of competency level for mastering educational knowledge.
- Conditions are created for students to be able to employ the theoretical knowledge they have acquired to various practical situations (including through the use of project work).

In our view, therefore, a promising route to the intellectual development of schoolchildren is through their work with specially constructed *developmental educational texts* which fulfil psycho-didactic requirements.

Psycho-didactic requirements for educational texts

We formulated *psycho-didactic requirements* for mathematical educational texts, fulfilled by developing corresponding textbooks and educational material as part of the MPI project.

1 A *thematic organisation* of mathematics course content (each textbook and study book is written on a specific topic from the curriculum of Years 5–9). For example, the textbook for Year 5 includes the topics "Natural numbers and decimals" and "Positive and negative numbers"; the textbook for Year 6 the themes "Solving equations", "The divisibility of numbers", "Rational numbers" and "Coordinates, graphs and symmetry".

   The thematic principle takes into account the possibility of a sequential development of topics, but also allows the implementation of an "immersion" teaching technique by deepening and broadening relevant material, thanks to the use of educational texts of various types (instructive, explanatory, narrative, historical-cultural and psychological).

2 *The multilevel structure of the educational text*, using various forms of presentation of educational information (verbal, visual, practical, emotional-metaphorical), various routines to master mathematical ideas (logical rationalisation, the analysis of real-life practical situations, using "unreal" aspects of mathematical knowledge), various sorts of educational activity (performance, research, project work, creative work) and various forms of self-monitoring.

3 *The interactive character of educational texts*. Educational texts are constructed as dialogues with the pupil-reader: they include various different forms of question on hypothetical problem situations, oriented towards discussion and debate of alternative points of view.

4 *An orientation towards comprehension* of mathematical facts and ideas. Educational texts are constructed taking into account the patterns of concept-forming processes (the development of various modes of information encoding, formation of cognitive schemes of mathematical concepts and methods of mathematical activity, work with the semantics of mathematical language, work with concept indicators, the establishment of various interconnections between concepts, the different stages in the formation of concepts, including the stage of motivation in the introduction of new mathematical concepts).

   Educational texts help the development of general intellectual ability (the ability to make an argument, evaluate, justify, plan, predict, react to contradictions, research and so on). The text of articles in the textbook is constructed according to the following model: first of all it ensures assimilation of the theoretical material, and only after this does the student move on to solving tasks in the practical workbook.

   Presentation of the material is sequential and "slow", with detailed discussion of various aspects of the mathematical object introduced. This allows the student to evaluate the gaps in his or her knowledge and the reasons behind it, to perceive the opportunity to solve the same task by a number of different means, and so on.

5 *Students' independent activity* in the process of acquiring new mathematical knowledge (the text "sets free" the student to go forward, allowing him or her independently to master a given question by him- or herself; a gradual transfer of the goal-setting function of educational activity to the student him- or herself and the stimulation of the student towards independent generation of educational text).

6 *Running diagnosis of the dynamics of students' educational-cognitive activity*. As the means of running testing in the educational texts various diagnostic material is used: multi-level educational tasks with different levels of difficulty (levels I and II); three options for controlled work relating to the students' preferred form of monitoring (of the type "Calculate …"; "Prove that …"; "Write an account of … and give examples"); "Test yourself" sections for self-testing of own knowledge and so on.

7 *Differentiation and individualisation of instruction* for students with various levels of previous education and various cognitive styles through diverse forms of presentation of the material, also taking into account their individual cognitive inclinations and preferences. Through the educational text the student is given the opportunity to choose how to master the material (through game-playing, performance, research, project work or creative activity), the degree of difficulty and various types of self-monitoring.

8 *Using the student's personal experience* by taking into account his or her everyday impressions, common-sense knowledge, willingness to trust his or her own intuitive judgements in the analysis of educational information.

9 *The creation of a psychologically comfortable regime for intellectual work*. What we mean by a psychologically comfortable regime for intellectual work is that type of education which evokes feelings of satisfaction and interest, leading to each student having a sense of success in his or her learning activity. In particular, narrative texts in study books (stories in which well-known figures from children's literature appear) help give emotional support to students with educational and personality problems (above all, those children who were unsuccessful in mathematics at primary school).

It is important to emphasise that the implementation of the psycho-didactic approach is necessary not just to improve the quality of learning of the school subject and create conditions for the intellectual growth of the student, but also to encourage a positive attitude towards different school subjects. This is especially urgent in the study of mathematics, since data show that children experience fear of mathematics, which is transformed into a negative attitude towards the mathematics teacher (Picker and Berry 2001).

We will try to illustrate the construction of educational texts on the example topic of "Quadratic equations", in particular the theory of the relation between the roots of a quadratic equation and its coefficients – Vieta's theorem. In order to see the difference in the way the educational material treats this topic it will suffice to analyse a small number of school textbooks.

One of the educational texts uses a story-telling approach and has the character of a reference book. The scientific fact (normative knowledge) is communicated at once: "For a reduced quadratic equation the correct theorem …" Thereafter examples of the application of the theorem are given.

Another educational text, which begins with a detailed review of a single example, is a typical example of an explanatory-illustrative text:

> The reduced quadratic equation $x^2-7x+10=0$ has the roots 2 and 5. The sum of the roots is equal to 7, and the product is equal to 10. We see that the sum of the roots is equal to the second coefficient, taken with the reverse sign, but the product is equal to the absolute term.

Hereafter the authors direct the students' attention to the fact that this property is inherent to any quadratic equation and then formulate and prove Vieta's theorem.

A third educational text on this theme, constructed according to psycho-didactic requirements, appears as follows:

> Studying quadratic equations, you will probably have noticed already that information about their roots is hidden in the coefficients. One or two 'secrets' have already become clear to us.
>
> The availability or absence of roots in a quadratic equation depends on the discriminant which is given by the coefficients of the equation. The roots of the equation can be found by a formula using the coefficients of the quadratic equation.
>
> How else are the roots and the coefficients of a quadratic equation connected? In order to uncover these connections it is useful to observe the coefficients and roots of various quadratic equations.
>
> Exercise 1. Solve the equations:
>
> $$x^2 + 5x + 6 = 0; \quad (22.1)$$
>
> $$x^2 - 5x + 6 = 0. \quad (22.2)$$
>
> Compare the coefficients of these equations, and then the roots. What connections between the roots and the coefficients do you notice in the equations?
>
> Do these equations support your findings:
>
> $$x^2 - 7x + 6 = 0; \quad (22.3)$$
>
> $$x^2 + 7x + 6 = 0; \quad (22.4)$$
>
> $$x^2 + 8x + 6 = 0; \quad (22.5)$$
>
> $$x^2 - x - 6 = 0? \quad (22.6)$$
>
> Try to formulate your findings and write them algebraically.
>
> When searching for patterns researchers often record their observations in tables (see Table 22.1). This helps to reveal these patterns. We suggest you also complete one of these tables.

	Equation	*p*	*q*	x_1	x_2	x_1+x_2	x_1-x_2
1	$x^2+5x+6=0$						
2	$x^2-5x+6=0$						
3	$x^2-7x+6=0$						
4	$x^2+7x+6=0$						
5	$x^2+8x+6=0$						
6	$x^2-x-6=0$						

> Has this table helped you to discover new connections between the roots and coefficients of quadratic equations?
>
> Does it make sense to include the standard form reduced equation $x^2+px+q=0$ in the table?

Compare your findings about the connections between the roots and coefficients of reduced quadratic equations with the findings shown in the following theorem.
[Thereafter follows the formulation of Vieta's theorem.]

Clearly the study of each of these educational texts acquaints students with a well-known scientific fact. However, students are brought to this normative knowledge by different routes; its acquisition is accompanied by different forms of intellectual activity. In our view, only the third educational text can be attributed to the category of developmental educational texts, insofar as it is organised as a dialogue with the student-reader, is oriented towards the comprehension of mathematical facts, promotes development of the ability to reason (analyse, compare, generalise, draw conclusions) and stimulates investigative activity.

A psycho-didactic typology of educational texts

Based on the structural model of intelligence (Figure 22.1) we have prepared developmental educational texts of various types for a school mathematics course (Years 5–9): each type of educational mathematical text conforms to a specific component in the structure of mental experience and is oriented towards its development (Gelfman *et al.*, 1996; Gelfman and Kholodnaya 1997, 1999).

In particular, we have presented in educational texts paths to enrichment of *components of cognitive experience* (the formation of various modes of information encoding, cognitive schemes, semantic structures), conceptual experience (taking into account patterns of concept acquisition), *components of metacognitive experience* (the development of involuntary and voluntary intellectual control, metacognitive awareness, an open cognitive position) and the *components of intentional (emotional-evaluative) experience* (the creation of conditions for the actualisation of cognitive preferences, convictions, attitudes of the students).

Table 22.1 shows a *psycho-didactic typology of educational texts* (using the example of mathematics educational texts).

It should be emphasised that each type of text (and here these are "microtexts") appears as an element of the whole text of the textbook or educational and methodical set, which corresponds to the above-formulated pyscho-didactic requirements for educational texts.

Conclusion

The trial use in schools of a new generation of mathematical textbooks and educational material, developed using the psycho-didactic approach, has an important consequence: what comes to the fore is the task of shaping the individual intellectual resources of school graduates; the level of development of these provides a base for their willingness to engage in innovative activity in future life, both in their professional work and in life as a whole.

Table 22.1 Psycho-didactic typology of educational texts

Forms of mental experience	*Components in the structure of mental experience*	*Characteristic features of educational-cognitive activity*	*Types of educational texts*
Cognitive experience	Modes of information encoding	Verbal-symbolic mode of information encoding	• text – mastering mathematical notation • text – looking for formulae • text – reaching formulation
		Visual mode of information encoding	• text – developing normative image • text – motivation for image • text – development of image • text – classification of image • text – activation of individual visual experience
		Practical mode of information encoding	• text – laboratory work • text – practical situation
		Sensory-emotional mode of information encoding	• text – emotional impression • text – metaphor • text – game
	Cognitive schemes	Cognitive schemes of mathematical concepts and methods of mathematical activity	• text – introduction of focus-example • text – framework • text – procedure • text – summary
	Semantic structures	Semantics of mathematical language	• text – the meaning of term • text – systematisation of the meaning of terms • text – translation from the language of mathematical symbols into the native language
	Conceptual structures	Patterns of mathematical concept formation	• text – microcomposition • text – identification of concept indicators • text – choice of concept indicators • text – establishing connections between concepts • text – motivation for concepts • text – categorisation • text – enrichment of the content of the concept • text – transferring the concept to a new situation • text – consolidation of content of concept

Table 22.1 Psycho-didactic typology of educational texts

Forms of mental experience	*Components in the structure of mental experience*	*Characteristic features of educational-cognitive activity*	*Types of educational texts*
Metacognitive experience	Involuntary and voluntary intellectual control	Planning	• text – programme • text – goal selection • text – constructing a plan • text – problematisation
		Prediction	• text – developing hypotheses • text – prediction in uncertain situations • text – predicting the result of a function
		Self-control	• text – methods of self-control • text – selection of methods of self-control • text – looking for errors
	Metacognitive awareness	Awareness of methods of mathematical activity, developing representations about one's intellectual resources	• text – reflection on solving methods • text – self-appraisal of one's knowledge and abilities • text – educational self-testing • text – independent composition of text • text – psychological commentary
	Open cognitive position	Readiness to work with contradictory information	• text – contradiction • text – alternative • text – conflict of opinions • text – impossible situation
Intentional (emotional-evaluative) experience	Preferences Convictions Attitudes	Choice of methods of study	• text – individual cognitive style • text – selection of methods of action • text – selection of cognitive position
		Actualisation of personal experience	• text – conjecture • text – creative work
		Value attitudes towards educational material	• text – history of mathematics • text – mathematics in the wider world • text – major lines in the development of mathematics

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