

## **Development-focused educational texts as a basis for learners' intellectual development in studying mathematics (DET technology)**

Marina A. Kholodnaya<sup>a\*</sup>, Emanuila G. Gelfman<sup>b</sup>

<sup>a</sup> Department of Psychology of Abilities and Mental Resources, Institute of Psychology Russian Academy of Science, Moscow, Russia

<sup>b</sup> Mathematics Teaching Department, Tomsk State Pedagogical University, Tomsk, Russia

\*Corresponding author. E-mail: kholod1949@yandex.ru

The article discusses an innovative teaching technology that uses development-focused educational texts (DET Technology) to stimulate school pupils' intellectual development in grades 5 to 9. It describes the psychological and psychodidactic framework of DET Technology. Development-focused educational texts are distinctive in that they use a framework of academic mathematical knowledge to build up the key components of pupils' mental experience (cognitive, conceptual, metacognitive, and intentional). Such texts also provide the conditions for the development and usage of students' personal learning styles. The article outlines the psychodidactic types of development-focused educational texts and the requirements that the psychodidactics proposes for educational texts.

**Keywords:** psychodidactics, intellectual development, development-focused educational text, mental experience, DET technology

### **Introduction**

People's intellectual abilities have started to be considered a key factor in social development due to the current challenges facing people today. Therefore, it is hardly surprising to observe a growing focus on school education and, above all, on innovative teaching technologies because the comprehensive school is a fundamental social institution that reproduces and enhances a society's intellectual resources.

The traditional, *subject-centered* teaching system assesses the effectiveness of pupils' education based primarily on their knowledge, capabilities, and skills in a particular school subject. In contrast, innovative teaching technologies that use the *psychodidactic* approach switch the main focus when evaluating teaching effectiveness to changes in a pupils' intelligence and personality that characterize them as being able to respond successfully to contemporary social challenges.

Psychodidactics is the area of pedagogy that designs content, forms, and methods of teaching based on integrated psychological, didactic, methodological and subject-matter knowledge while focusing specifically on the mental patterns of personality development as a basis for organizing the teaching process and general learning environment (Davydov, 1966; Panov, 2004; Gelfman and Kholodnaya, 2006; et al.).

Psychodidactic efforts result in a qualitatively new teaching product that combines psychological, didactic, methodological and subject-matter knowledge. That product could be a new schooling environment, an innovative teaching technology, a development-focused training method or a next-generation textbook. The psychodidactic approach involves *pedagogical engineering*, i.e., the design, structuring and use of teaching products that focus specifically on developing the mental resources of each child. The main goal of psychodidactics is to provide an environment that promotes children's psychological growth by teaching a given subject more effectively.

The psychodidactic approach may be used in schools in many ways, e.g.:

- by using "didactical situations" as a teaching technique to shape pupils' knowledge, including the use of metaphor and emotional context (Brousseau 1997);
- through focusing on learning and conceptualization by selecting mathematical tasks and hypothesizing about how each one influences the learning process (*hypothetical learning trajectory*, or *HLT*) (Simon, Tzur, 2004);
- by using basic cognitive actions such as recognizing, building-with and constructing (RBC model) as a foundation for conceptual teaching based on pupils' own experience (Hershkowitz et al., 2001; Bikner-Ahsbahs 2004);
- by developing pupils' creative thinking (Burke and Williams 2008); and
- by using "realistic" situations in the learning process (RME) (Van den Heuvel-Panhuizen & Drijvers, 2014).

The actual content of school subjects is essential to pupils' intellectual development. It is therefore very important to set the requirements applied to the teaching content. This includes school textbooks that could be used to implement the psychodidactic approach.

We believe that a textbook should not be structured as a reference/problem book. Rather, it should be a learner-focused teaching book because mathematical knowledge can have a developmental effect only when it is in harmony with the patterns of pupils' mental development (intellectual and personal). Moreover, the work that learners do with the teaching texts—e.g., their own analysis, identification of key ideas, interpretation, and text writing—is a powerful resource for intellectual development. Ignoring the option of this type of work would mean denying pupils the opportunity to do "quiet" intellectual work on their own.

We developed *Development-focused Educational Texts (DET) Technology* as part of the *Mathematics. Psychology. Intelligence (MPI)* pedagogical project for use in teaching middle-school mathematics (years 5 to 9). DET Technology focuses on pupils' intellectual development based on *mathematics content* and special-purpose *educational texts* (Gelfman and Kholodnaya 2006; Kholodnaya, Gelfman 2016).

This article explains the psychological and psychodidactic foundation of DET Technology and outlines the requirements applied to educational texts that focus on pupils' intellectual development.

### **Educational texts as a way to foster pupils' intellectual development**

When assessing the role of school education content as a whole and the role of educational texts in particular, one must appreciate the key fact that from the psychological perspective, intellectual development is possible only through learning, processing and producing diverse subject content, ranging from trivial everyday knowledge to scientific hypotheses about the structure of the universe. The richer the subject-matter environment (physical, social and educational) surrounding a preschooler or schoolchild, and the more actively they interact with this environment, the greater their intellectual ability will be. Moreover, the quality of subject content is fundamentally important. The developmental effect will differ depending on whether the learner studies by reading only normative texts or by reading texts that are open and thought-provoking and whether she/he learns mathematics by solving many similar tasks or by doing her/his own research.

Therefore, *the content of school education* is a key factor in the development of pupils' intelligence. Its minimum component is *the educational text*, which shapes the way pupils interact with various content environments.

In a wider sense, *the educational mathematical text* is any set of signs and symbols from the language of mathematics or the natural (Russian) language that carries a mathematical meaning. Consequently, educational mathematical texts range from detailed theoretical descriptions to individual tasks (e.g., problems, formulas, diagrams, drawings).

The text is the most valuable culture element and is essential for teaching. The important role of texts in personal intellectual development is noted by many scholars. They consider the text to be "a thinking structure" (V.V. Ivanov), "a model of thought adventures" (L.E. Gendenshtein) and "a conversation partner" (M.M. Bakhtin). Texts are a natural medium for intellectual development throughout a person's lifetime. In a wider sense, a "text" is a message that a person has to read and interpret about, including everyday situations, natural phenomena, and the behavior of others.

Texts, whether scientific, historical, cultural, fictional or educational, are not linear. An educational text should be designed as a multi-dimensional semantic space allowing the reader to travel mentally in different directions.

Development-focused educational texts are a type of *hypertext* characterized by the following:

- *Nonlinearity*. The "nucleus" or subject-matter knowledge in development-focused texts is supported by transitions from normative text fragments to topical discussion elements, including non-subject-related texts (stories, play elements, practical physics, ecology and psychology knowledge).
- *Diversity*. These texts contain elements with differing degrees of complexity in both content and learning methods (e.g., texts and tasks of various difficulty levels, normative texts containing templates and open texts, learning

through performance, research, projects and creative work). They employ various genres and types of texts (e.g., narrative, explanatory, discursive, problem-focused, "impossible") and involve numerous ways of presenting knowledge (verbal/logical, visual, substantive/practical, emotional/meta-phorical).

- *Incompleteness, ambiguity and contradiction.* The educational texts contain an element of uncertainty, which is increased by questions addressed to the learner. The main benefit of this is that it stimulates a desire to eliminate uncertainty, which is a key aspect of creativity.
- *Personal involvement.* This effect is created through dialogue (including questions to the reader); structuring the text as a narrative—which is essential for supporting pupils emotionally and harnessing their everyday experiences—and by offering a selection of ways to study depending on the pupil's level of training, preferences and individual learning style. Pupils should be able to move on to the deeper levels in the text and recover non-verbal implications by relying on words' connotative meanings and their personal experience, associations and imagination.
- *Navigation (migration) across the text.* Learners can transfer from textbook texts to narrative teaching books, workbooks or electronic materials. In the workbook, they can undertake tasks of different complexity levels and use computer software to self-check and develop the required skills. The textbooks contain navigation tools (a navigation bar with special icons) that enable pupils to use the teaching set components according to their learning needs.
- *Self-sufficiency.* The educational texts are constructed in a way that knowledge is never easily available. Pupils need to approach new concepts and definitions gradually on their own while acquiring the necessary self-checking skills (e.g., willingness to pause, reflect and then possibly spot a mistake in the discourse). Texts that encourage pupils to write their own content unaided play an important role.

Therefore, next-generation educational texts are development-focused texts that provide teachers with a variety of learning trajectories depending on the child's needs.

In school education, texts are always the focus of attention because of their essential role in effective teaching (Eason et al., 2012; Geary, 2011; Siegler et al., Osmolovskaya, 2014, et al.). "Reader-oriented theory" suggests that readers actively construct meanings (concepts) as they read, which also applies to reading mathematics textbooks (Weinberg and Wiesner 2011).

Noting the importance of educational texts in school mathematics, we can push the limits of the popular view that teaching mathematics simply means teaching pupils how to solve mathematical problems. We believe that teaching mathematics actually means teaching children how to interpret the meanings and implications of mathematical concepts and operations. By working on educational texts, pupils can achieve a high level of understanding, which, in turn, will enable them to solve problems.

Therefore, the use of special *development-focused educational texts* that meet specific psychodidactic requirements is a promising way to encourage intellectual development. It means that development-focused educational texts do not merely present formal mathematical knowledge; they facilitate the development of psychological mechanisms for productive intellectual activity.

The question that needs to be answered is what should be the psychological basis of development-focused mathematical texts so they can boost pupils' intellectual development.

### **Enrichment of mental experience as the psychological basis for pupils' intellectual development**

We believe that the psychological basis for intellectual development should be the enrichment of pupils' mental experiences while learning.

The structural model of intelligence, as viewed in terms of the architecture of a person's mental experience, outlines four levels of mental experience, each with its specific purpose (Kholodnaya 2002; 2004):

- 1) *Cognitive experience* refers to the mental structures ("cognitive schemes") responsible for presenting, recognizing, storing and sorting information. Their main role is immediate information processing.
- 2) *Conceptual experience* refers to mental structures ("concepts") that generalize and transform information through abstraction, idealization and interpretation. Their main purpose is to identify meaningful properties and reproduce regular and consistent features of the environment.
- 3) *Metacognitive experience* refers to mental structures ("metacognitions") that allow involuntary and voluntary regulation of information processing and conscious control over how intelligence works. Their main purpose is to control intellectual activity and the status of personal intellectual resources.
- 4) *Intentional (emotional and evaluative) experience* refers to the mental structures ("intentions") underlying individual cognitive dispositions. The main purpose of these structures is to form subjective preferences in selecting, e.g., subject areas, ways of solving problems, and information sources.

The way cognitive, conceptual, metacognitive and intentional experience is organized defines *the productive properties* of an individual's intelligence (cognitive, conceptual, metacognitive, creative abilities) and the *stylistic properties* (information-encoding styles, information-processing styles, thinking styles and epistemological styles). As intelligence develops, its productive and stylistic properties provide a foundation for integral intellectual abilities, including competence, talent and wisdom, which are the forms of intellectual giftedness.

If we define intelligence as the profile of an individual's mental experience, then we can suggest that each pupil brings his own mental experience that predetermines the nature of his intellectual activity in a given situation. The content and structure of this mental experience differ from one pupil to the next, which means that children's intellectual ability will vary. However, they all need an environment

that can facilitate their intellectual development through maximal enrichment of their personal mental experience.

“Enrichment” here includes, first, the development of each pupil’s key mental experience components (such as cognitive, conceptual, metacognitive and intentional experience) as a foundation for nurturing their intellectual abilities, Second, it establishes the conditions for pupils to demonstrate their individual cognitive styles.

In short, we can summarize the task of boosting pupils’ intellectual development using the following propositions:

- Each pupil carries his own mental experience and therefore has certain initial intellectual resources. Due to the structural and content distinctiveness of each pupil’s mental experiences, each individual is “clever in his own way”.
- In school education, pedagogical efforts target the content and structure of mental experience, including its cognitive, conceptual, metacognitive and intentional components.
- The mechanisms of an individual’s intellectual growth are linked to processes within that person’s mental experience and arise when experience components are sophisticated and enriched.
- Each pupil has her own potential for strengthening intellectual abilities, and the teacher’s task is to provide the necessary support by customizing classroom and after-school activities to their abilities.

### **Psychodidactic typology of educational texts**

*DET Technology* suggests that efforts to enrich pupils’ mental experience through development-focused educational mathematical texts should mainly focus on the following (Gelfman, Kholodnaya 2006; Kholodnaya, Gelfman, 2016):

- *Enrichment of cognitive experience.* This should seek to develop the use of different ways of information encoding (verbal/symbolic, visual, substantive/practical, sensory/emotional), to widen the range of declarative and procedural cognitive schemes for mathematical concepts and activity methods and to increase their flexibility.
- *Enrichment of conceptual experience.* This involves improving students’ understanding of mathematical language semantics, expanding the semantic fields pertaining to mathematical concepts, and differentiating and integrating verbal and non-verbal semantic structures. It also focuses on helping students identify substantial conceptual features and links between concepts from different generalization levels. It relies on independent concept building based on hypothesizing. Moreover, it takes into account the main phases of conceptualization, such as motivation, categorization, enrichment, transfer and crystallization.
- *Enrichment of metacognitive experience.* This enrichment should help develop voluntary and involuntary control of intellectual activity, including the ability to plan, evaluate, predict, and self-check. . Its aim is to increase

*metacognitive awareness*—in other words, the student's understanding of how academic knowledge is organized and differences between learning methods. It encourages an *open cognitive position*, which means that it encourages a readiness, e.g., to absorb “impossible” information, accept an alternative point of view, and properly react to discrepancies.

- *Enrichment of intentional (emotional and evaluative) experience.* This means offering students *a choice* of how to study educational materials. It relies on *the pupil's personal and intuitive experience*. (Students are encouraged to share doubts, guesses, beliefs, “anticipatory” ideas and emotional evaluations.) *Play elements* are used, and there is a *value-based approach* to educational materials. The efforts should also include promoting multiple individual cognitive styles that reflect personal preferences and dispositions.

Based on the structural model of intelligence, different development-focused educational texts were designed for the school mathematics courses (years 5 to 9). Each type of mathematics teaching text addressed a specific component of the mental experience framework with the aim of facilitating its development (Gelfman and Kholodnaya 2006; Kholodnaya and Gelfman, 2016).

Table 1 describes the *psychodidactic typology of development-focused educational texts* (using the example of mathematics teaching texts).

**Table 1.** Psychodidactic typology of development-focused educational texts

Forms of mental experience	Components of mental experience structure	Characteristics of educational activity	Types of development-focused educational texts
Cognitive experience	Information-encoding ways	Verbal/symbolic information encoding	<ul style="list-style-type: none"> <li>• learning mathematical symbols</li> <li>• finding a formula</li> <li>• drafting definitions</li> </ul>
	Visual information encoding		<ul style="list-style-type: none"> <li>• developing a normative image</li> <li>• image classification</li> <li>• image evolution</li> <li>• new image motivation</li> <li>• conversion from verbal/symbolic to visual encoding</li> <li>• initiation of personal imaginative experience</li> </ul>
	Substantive/practical information encoding		<ul style="list-style-type: none"> <li>• laboratory work</li> <li>• situation in practice</li> </ul>
	Sensory/emotional information encoding		<ul style="list-style-type: none"> <li>• emotional impression</li> <li>• metaphor</li> <li>• play</li> </ul>
	Declarative cognitive schemes	Cognitive schemes of mathematical concepts	<ul style="list-style-type: none"> <li>• introduction of focus example</li> <li>• frame of concept</li> <li>• summary</li> </ul>
	Procedural cognitive schemes	Cognitive schemes of mathematical activity methods	<ul style="list-style-type: none"> <li>• algorithm (procedure)</li> <li>• operation</li> </ul>

Table 1 (concluded)

Forms of mental experience	Components of mental experience structure	Characteristics of educational activity	Types of development-focused educational texts
Conceptual experience	Semantic structures	Mathematical language semantics	<ul style="list-style-type: none"> <li>• term meaning</li> <li>• systematization of term meanings</li> <li>• translation from the language of mathematical symbols to native language</li> </ul>
	Category structures	Identification of category features and establishment of links between categories	<ul style="list-style-type: none"> <li>• identification of concept features</li> <li>• selection of concept features</li> <li>• establishment of links between concepts</li> <li>• concept motivation</li> <li>• concept categorization</li> <li>• enrichment of conceptual content</li> <li>• transfer of a concept to a new situation</li> <li>• crystallization of conceptual content</li> </ul>
	Generative structures	Concept building and the creation of own texts; interpretation and modeling	<ul style="list-style-type: none"> <li>• searching for and generalizing regularities</li> <li>• micro essay</li> <li>• modeling</li> <li>• independent writing of an original text</li> <li>• the invitation to the project</li> </ul>
	Involuntary and voluntary intellectual control	Planning	<ul style="list-style-type: none"> <li>• program</li> <li>• goal selection</li> <li>• planning</li> </ul>
		Predicting	<ul style="list-style-type: none"> <li>• developing a hypothesis</li> <li>• prediction in an uncertain situation</li> <li>• predicting operation results</li> </ul>
		Self-checking	<ul style="list-style-type: none"> <li>• self-checking methods</li> <li>• choosing a self-checking method</li> <li>• search for mistakes</li> </ul>
Metacognitive experience	Metacognitive awareness	Understanding methods of mathematical activity and own intellectual resources	<ul style="list-style-type: none"> <li>• reflection on solution methods</li> <li>• self-assessment of personal knowledge and skills</li> <li>• educational self-monitoring</li> <li>• psychological commentary</li> </ul>
	Open cognitive position	Readiness to work with inconsistent information	<ul style="list-style-type: none"> <li>• problematization</li> <li>• alternative</li> <li>• contrasting opinions</li> <li>• impossible situation</li> </ul>
	Preferences Beliefs Attitudes	Choice of learning method	<ul style="list-style-type: none"> <li>• selection of activity methods</li> <li>• selection of cognitive position</li> <li>• initiation and formation of individual cognitive styles</li> </ul>
		Use of intuitive experience	<ul style="list-style-type: none"> <li>• conjecture</li> <li>• creative work</li> </ul>
		Value-based treatment of educational material	<ul style="list-style-type: none"> <li>• mathematics and the wider world</li> <li>• key directions in mathematics</li> <li>• development</li> <li>• history of mathematics</li> </ul>
Intentional (emotional and evaluative) experience			

It should be stressed that each text type in Figure 1 is a type of prototype text with an invariable characteristic (its focus on a certain mental experience component) and variable semantic characteristics that depend on the curriculum topic. Moreover, texts of each type could serve as microtexts within a larger educational text; therefore, one text can contain several types of educational texts.

### **Psychodidactic requirements applied to development-focused educational texts**

The *Mathematics. Psychology. Intelligence. (MPI)* project and *DET Technology* offer the psychodidactic requirements for mathematics teaching using development-focused educational texts that form the content of textbooks, problem books, workbooks and electronic materials used to teach pupils in school years 5–6 and 7–9.

The proposed requirements are:

- 1) *Topic-based structure of mathematics course.* Each textbook/teaching book is devoted to a specific topic. For example, in year 5, the topics are ‘Natural numbers and decimals’ and ‘Positive and negative numbers’. In year 6, the topics include ‘Solving equations’, ‘Divisibility of numbers’, ‘Rational numbers’, and ‘Coordinates. Diagrams. Symmetry’. The same principle of content arrangement is used in the textbooks for years 7–9. Topic-based teaching allows the topic to be developed consistently and the “immersion” teaching technique to be used with the help of teaching material on a deep, broad level using a variety of text types (instructions, explanations, stories, and history and psychology texts).
- 2) *Polymodality.* Texts provide multiple approaches to information presentation (verbal/logical, visual, substantive/practical, emotional/metaphorical), different methods of learning mathematical concepts (rationalization, case studies, use of “impossible” aspects of mathematics), different forms of learning activities (performance, research, project-based and creative work) and different self-check methods.
- 3) *Integration of declarative, procedural, metacognitive and value knowledge.* In addition to systematizing mathematical concepts, special emphasis is placed on teaching pupils how to learn effectively and efficiently. Pupils are taught (including through project assignments) how to understand algorithms, to solve text-based problems, to choose a rational solution method, and to analyse one and the same problem in a variety of ways, among other skills. Moreover, certain texts give pupils information about their own intellectual qualities, about the way academic knowledge is structured and about how they can acquire it. Finally, the texts describe possible attitudes towards facts, phenomena, actions, and conclusions, which helps stimulate pupils’ ability to express their own judgment.
- 4) *Focus on understanding mathematical facts and ideas.* The educational texts rely on the psychology underlying conceptualization. Their aim is to promote different information-encoding methods, to establish cognitive schemes pertaining to mathematical concepts and operations, to work with

the semantics of mathematical language, and to find concept features and links between concepts. They also reflect the stages of conceptualization; in particular, they encourage the introduction of new mathematical concepts. These educational texts facilitate the development of the general intellectual skills that are involved in understanding, such as the ability to argue, evaluate, substantiate, plan, predict and react correctly to inconsistencies and research.

- 5) *Development of a reflective position.* The content of teaching materials—i.e., the structure of each topic and the questions and tasks provided—is arranged to promote reflection among students or a conscious, voluntary attitude towards the learning process. The materials present knowledge gradually and slowly, providing a detailed exploration of the various aspects of the mathematical object introduced. This enables pupils, e.g., to evaluate gaps in their knowledge and the reasons behind them and to understand how different methods can be used to solve the same problem.
- 6) *Dialogue.* Development-focused educational texts are designed as dialogues with the reader. They include questions relating to topical situations and encourage discussions of alternative viewpoints. They also teach students how to express, substantiate and defend their opinion. Story-based educational books have a special place in DETs since the dialogue involves pupil participation in analyzing mathematical problems together with the story characters.
- 7) *Pupils' self-sufficiency* at different stages of mathematical studies. The texts give readers a chance to move forward for a while, letting them acquire an amount of knowledge on their own. They also use the method of speaking to the reader to cause pupils to act independently. Gradually, pupils begin to set themselves learning goals independently and are encouraged to produce their own educational texts.
- 8) *Customizing training* for different levels of competence and learning styles is achieved by presenting teaching material in different ways while taking individual learning appetites and preferences into account. DET texts offer pupils a choice of how they want to learn, be it through play, performance, research, projects or creation. They can also choose the difficulty level of the material they want to learn and what tasks they want to perform to test their knowledge. The versatile structure of the DET teaching-aid sets respects pupils' personal learning needs and preferences and allows them to choose a customized learning trajectory.
- 9) *Reliance on personal experience* takes students' everyday impressions and knowledge into account and their readiness to trust their intuitive judgment when analyzing information during theoretical learning and problem solving. In particular, certain DET texts provide practical information designed to stimulate pupils' interests in everyday applications of mathematics while demonstrating the role of mathematics in real life.
- 10) *Controlling the learning dynamics.* DET texts provide materials for routine progress checks, such as tasks of varying complexities (Levels I, II and III),

three test options depending on the preferred control method (calculation, proving, writing a story with examples), and self-check sections.

- 11) *Psychologically comfortable conditions for intellectual activity.* When conditions for intellectual activity are comfortable from the psychological perspective, they promote enjoyable and interesting learning and help each pupil feel that his learning efforts are effective. Narrative texts, for example, involve book characters who are known to pupils and support pupils emotionally with learning-related and personality issues; most importantly, they encourage pupils who had difficulties studying mathematics in primary school.

It is important to note that the psychodidactic approach does not merely improve the quality of learning and promote intellectual development; it also helps to establish a positive attitude towards a school subject. The approach is especially relevant for mathematics. There are data suggesting that children can experience a fear of mathematics that translates into a negative attitude towards the mathematics teacher (Picker and Berry 2001).

Here is an example of a development-focused educational text from the year 8 textbook. It includes types of text such as “*problematization*”, “*finding a formula*”, “*drafting definitions*”, and “*searching for and generalizing regularities*” (Gelfman et al. 2013, p. 114-115).

“\\$25. Studying the connection between roots and coefficients in a quadratic equation. Vieta’s theorem.

You have probably already noticed that the information about the nature of the roots in a quadratic equation is hidden in its coefficients. This is no longer a secret for us.

Whether a quadratic equation has any roots or not depends on the sign of the discriminant, which is a function of the equation’s coefficients.

An equation can be solved using a formula that includes its coefficients.

In what other ways are the roots and coefficients of quadratic equations connected? In order to understand these connections, it is useful to study how coefficients and roots in different equations behave.

Task 1. Solve the quadratic equations:

a)  $x^2 + 5x + 6 = 0$ ;      b)  $x^2 - 5x + 6 = 0$ .

What relations have you noticed between the roots and coefficients of these equations?

Are your conclusions correct for the equations below?

c)  $x^2 - 7x + 6 = 0$       d)  $x^2 + 7x + 6 = 0$ ;  
e)  $x^2 + 6x + 8 = 0$ ;      f)  $x^2 - x - 6 = 0$ ?

Please try to formulate your conclusions and write them down algebraically.

When trying to find regular patterns, scholars often record their observations in tables, making it easier to discover such patterns. We recommend you to complete such a table too.

No.	Equation	P	q	$x_1$	$x_2$	$x_1+x_2$	$x_1 \cdot x_2$
1	$x^2+5x+6=0$	5	6	-2	-3	-5	6
2	$x^2-5x+6=0$						
3	$x^2-7x+6=0$						
4	$x^2+7x+6=0$						
5	$x^2+6x+8=0$						
6	$x^2-x-6=0$						
7	$x^2+px+q=0$						

Has the table helped you to understand the connection between the roots and coefficients of quadratic equations?

Compare your own conclusions with the ones provided in the textbook."

Therefore, the educational text above is aimed at enriching pupils' conceptual experience. It encourages pupils to recognize a problem, independently analyze the provided equations and generally define the substantial patterns and connections between mathematical objects (the coefficients and roots of a reduced quadratic equation).

DET technology, which is used to develop different types of educational textbooks taking into account the training requirements for the psychodidactic texts, is implemented in the educational project "Mathematics. Psychology. Intelligence" (MPI). The development-focused educational texts are presented in the form of a training package (or educational-methodical complex):

- Set for school years 5 and 6: textbooks, story-based teaching books, problem books, self-study workbooks, electronic materials (exercises, tests, cartoons, mathematics games, electronic guide), course program and training manual for the teacher;
- Set for school years 7, 8 and 9: textbooks, problem books, course program and training manual for the teacher.

Teaching mathematics in grades 5 to 9 according to the MPI training package has been carried out in schools in different Russian cities over past 10 years.

Our studies use psychodiagnostic techniques—in comparison to the remedial, traditional and enriching models—and show that systematic work with developing educational texts in studying mathematics (enriched learning model) leads to increased productivity of intellectual activity. An increase in intelligence and creativity is observed from grades 5 to 9. Field-independent, reflective, fast, accurate, and categorial strategies of information processing increase (in terms of indicators of cognitive styles of field dependence/field independence, impulsivity/reflectivity, narrow/wide range of equivalence) (Budrina 2009, 2010). Remarkably, through the enriching learning models, students show significant cumulative growth in the convergent, divergent and style properties of intelligence, which are most pronounced starting from grade 8 (in comparison with the traditional model).

## Conclusion

DET Technology provides a tool that encourages pupils' intellectual development by offering a set of development-focused educational texts on a variety of topics within a school mathematics course. Such texts facilitate the enrichment of the main components in pupils' mental experience and make it possible to customize teaching.

The use of special-purpose educational texts based on the psychodidactic approach to teaching promotes the development of intellectual resources that are instrumental in achieving successful school learning.

## Acknowledgements

This study was supported by a grant from the Russian Science Foundation (project № 14-28-00087), Institute of Psychology of Russian Academy of Sciences.

## References

- Biker-Ahsbahs, A. (2004). Towards the emergence of constructing mathematical meaning, In *Proceeding of the 28th Conference of the International Group for the Psychology of Mathematics Education*, 2, 119–126.
- Brousseau, G. (1997). *Theory of didactical situation in mathematics*. Dordrecht, The Netherlands: Kluwer.
- Budrina, E. G. (2009). Dinamika intellektualnogo razvitiya podrostkov v usloviyach raznuch modelei obucheniya [The dynamics of intellectual development of adolescents in different models of learning]. *Psychological Journal*, 30(4), 33–46.
- Budrina, E. G. (2010). Spezifika intellektualnogo razvitiya podrostkov v raznuch modeliach obucheniya [The specificity of the intellectual development of adolescents in different models of learning]. *Experimental psychology*, 3(1), 115–130.
- Burke, L. A., & Williams, J. M. (2008) Developing young thinkers: An intervention aimed to enhance children's thinking skills. *Thinking Skills and Creativity*, 3, 104–124. doi: 10.1016/j.tsc.2008.01.001
- Davydov, V. V. (1996). *Teoriya razvivauschego obrazovaniya* [The theory of developmental education]. Moscow: INTOR.
- Eason, S. H., Goldberg, L. F., Young, K. M., Geist M. C. & Cutting L. E. (2012). Reader-text interactions: How differential text and question types influence cognitive skills needed for reading comprehension. *Journal of Educational Psychology*, 104(3), 515–528. doi: 10.1037/a0027182
- Geary, D. C. (2011). Cognitive predictors of achievement growth in mathematics: A 5-year longitudinal study. *Developmental Psychology*, 47(6), 1539–1552. doi: 10.1037/a0025510
- Gelfman, E. G., & Demidova, N. L. (2013). *Algebra: Uchebnik dlya 8 klassa* [Algebra. Year 8 textbook]. Moscow: BINOM.
- Gelfman, E. G., & Kholodnaya, M. A. (2006). Psichodidaktika shkolnogo uchebnika: Intellektualnoe vospitanie shkolnikov [*Psycho-didactics of school textbooks: The intellectual nurture of students*]. St. Petersburg, Russia: Piter.
- Gelfman, E., Kholodnaya, M., & Cherkassov, R. (1997). From didactics of mathematics to psycho-didactics. In N.A. Malara (Ed.). *International view on didactics of mathematics as a scientific discipline. Proceedings WG25, ICME-8* (pp. 102–107). University of Modena, Italy.

- Hershkowitz, R., Schwarz, B., & Dreyfus, T. (2001). Abstraction in context: Epistemic actions. *Journal for Research in Mathematics Education*, 32, 195–222. doi: 10.2307/749673
- Kholodnaya, M. A. (2002). *Psichologiya intellekta: Paradozu issledovaniya* [The psychology of intelligence: Paradoxes of research]. St Petersburg, Russia: Piter.
- Kholodnaya, M. A. (2004). *Kognitivnye stili: O prirode individualnogo umia* [Cognitive styles: On the nature of the individual mind]. St. Petersburg, Russia: Piter.
- Kholodnaya, M. A., & Gelfman, E. G. (2016). *Razvivaushie uchebnue tekstu kak sredstvo intellektualnogo vospitaniaya shkolnikov* [Development-focused educational texts as a means for learners' intellectual development]. Moscow: Institute of Psychology, Russian Academy of Sciences.
- Love, E., & Pimm, D. (1996). "This is so": A text on texts. In A. J. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.). *International handbook of mathematics education* (Vol. 1, pp. 371–409). Dordrecht: Kluwer. doi: 10.1007/978-94-009-1465-0\_12
- Osmolovskaya, I. M. (2014). Uchebniki novogo pokoleniya: Poisk didakticheskikh reshenii [Textbooks of new generation: Search for didactic decisions]. *Otechestvennaya I zarubezhnaya pedagogika* [National and Foreign Pedagogy], 4(19), 45–53.
- Panov, V. I. (2007). *Psychodidaktika obrazovatelnykh sistem: Teoriya i praktika* [Psychodidactics of educational systems: Theory and practice]. St Petersburg, Russia: Piter.
- Picker, S. H., & Berry, J. S. (2001). Investigating pupils' images of mathematicians. In *Proceedings of the 25th Conference of the International group for the Psychology of Mathematics Education* (Vol. 4, pp. 49–56). Utrecht University, the Netherlands.
- Siegler, R. S., Duncan, G. J., Davis-Kean, P. E., Duckworth, K., Claessens, A., Engel, M., Susperreguy, M. I., & Chen, M. (2012). Early predictors of high school mathematics achievement. *Psychological Science*, 23(7), 691–697. doi: 10.1177/0956797612440101
- Simon, M., & Tzur, R. (2004). Explicating the role of mathematical tasks in conceptual learning: An elaboration of the Hypothetical Learning Theory. *Mathematical Thinking and Learning*, 6(2), 91–104. doi: 10.1207/s15327833mtl0602\_2
- Van den Heuvel-Panhuizen, M., & Drijvers, P. (2014). Realistic mathematics education. In S. Lerman (Ed.) *Encyclopedia of Mathematics Education* (pp. 521–525). Dordrecht, Heidelberg, New York, London: Springer. doi: 10.1007/978-94-007-4978-8\_170
- Weinberg, A., & Wiesner, E. (2011). Understanding mathematical textbooks through reader-oriented theory. *Educational Studies in Mathematics*, 76, 49–63. doi: 10.1007/s10649-010-9264-3

Original manuscript received February 15, 2016

Revised manuscript accepted April 1, 2016

First published online September 30, 2016